International Journal of Theoretical Physics, Vol. 45, No. 9, September 2006 (© 2006)
DOI: 10.1007/s10773-006-9139-7

New Types of Localized Coherent Structures in the Bogoyavlenskii-Schiff Equation

Yan-ze Peng[1]

Received October 9, 2005; Accepted March 13, 2006
Published Online: June 7, 2006

Based on the singular structure analysis, we derive some new types of localized coherent structures for the Bogoyavlenskii-Schiff equation by suitably utilizing the arbitrary function present in the singular manifold equations.

KEY WORDS: The singular structure analysis; the localized coherent structure; the Bogoyavlenskii-Schiff equation.

PACS: 05.45. Yv, 02.30.Ik, 02.30.Jr.

The Bogoyavlenskii-Schiff (BS) equation (Bogoyavlenskii, 1990; Schiff, 1992) is a $(2+1)$-dimensional nonlinear equation having the form

$$4u_{xt} + 8u_x u_{xy} + 4u_y u_{xx} + u_{xxxy} = 0. \tag{1}$$

Recently, Toda *et al.* (1999) obtained the BS hierarchy of Eq. (1) by the recursion operator. This hierarchy is reduced to the KdV hierarchy by setting $y = x$. In this paper, we will derive some new types of localized coherent structures for the BS equation (1). Truncating the Painlevé expansion of Eq. (1) at the constant level term yields

$$u = \varphi^{-1} u_0 + u_1, \tag{2}$$

where $\varphi \equiv \varphi(x, y, t)$ is the singular manifold. Substituting Eq. (2) into Eq. (1) and equating the coefficients of like powers of φ, we obtain

$$u_0 = \varphi_x, \tag{3}$$

where φ satisfies the system of equations

$$\begin{aligned} &4\varphi_t\varphi_x^2 + 4u_{1y}\varphi_x^3 + 8u_{1x}\varphi_y\varphi_x^2 - 2\varphi_x\varphi_{xy}\varphi_{xx} \\ &-\varphi_y\varphi_{xx}^2 + 2\varphi_x^2\varphi_{xxy} + 2\varphi_y\varphi_x\varphi_{xxx} = 0, \end{aligned}$$

[1] Department of Mathematics, Huazhong University of Science and Technology, Wuhan 430074, P. R. China; e-mail: yanzepeng@163.com

$$-8\varphi_x\varphi_{xt} - 16u_{1x}\varphi_x\varphi_{xy} - 8u_{1xy}\varphi_x^2 - 4\varphi_t\varphi_{xx}$$
$$-12u_{1y}\varphi_x\varphi_{xx} - 8u_{1x}\varphi_y\varphi_{xx} - 4u_{1xx}\varphi_y\varphi_x + 2\varphi_{xx}\varphi_{xxy} \tag{4}$$
$$-4\varphi_x\varphi_{xxxy} - \varphi_y\varphi_{xxxx} = 0,$$
$$8u_{1xy}\varphi_{xx} + 4u_{1xx}\varphi_{xy} + 4\varphi_{xxt}$$
$$+8u_{1x}\varphi_{xxy} + 4u_{1y}\varphi_{xxx} + \varphi_{xxxxy} = 0,$$

with u_1 satisfying the original equation (1). And u_1 is called the seminal solution of Eq. (1). Eqs. (2), (3) and (4) constitute an auto-Bäcklund transformation of the BS equation (1) in terms of the singular manifold. In order to obtain the exact solution of Eq. (1), we take the seminal solution $u_1 = 0$. Thus, Eq. (4) is simplified into

$$4\varphi_t\varphi_x^2 - 2\varphi_x\varphi_{xy}\varphi_{xx} - \varphi_y\varphi_{xx}^2 + 2\varphi_x^2\varphi_{xxy} + 2\varphi_y\varphi_x\varphi_{xxx} = 0,$$
$$-8\varphi_x\varphi_{xt} - 4\varphi_t\varphi_{xx} + 2\varphi_{xx}\varphi_{xxy} - 4\varphi_x\varphi_{xxxy} - \varphi_y\varphi_{xxxx} = 0, \tag{5}$$
$$4\varphi_{xxt} + \varphi_{xxxxy} = 0,$$

which is called the singular manifold equations. It is found that Eq. (5) has the variable separation solution of the form

$$\varphi = \mathrm{e}^x + g(4y - t), \tag{6}$$

where g is an arbitrary function of indicated variable. Substituting Eqs. (3) and (6) into (2) with $u_1 = 0$, one obtains a functional separation solution of the BS equation (1)

$$u = \frac{\mathrm{e}^x}{\mathrm{e}^x + g(4y - t)}. \tag{7}$$

It is easy to see that the BS equation (1) possesses some special types of localized coherent structures for the following potential field

$$w \equiv u_y = \frac{4\mathrm{e}^x g'(4y - t)}{[\mathrm{e}^x + g(4y - t)]^2}, \tag{8}$$

rather than the physical field u itself. Thanks to the arbitrariness of the function g, we may obtain some interesting localized coherent structures from Eq. (8) by choosing appropriately the arbitrary function g. Several cases are considered in what follows.

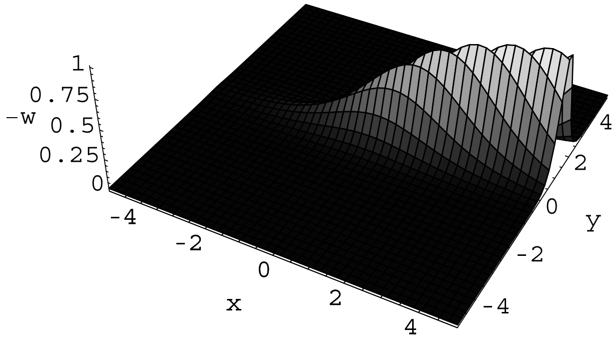

Fig. 1. The structure graph of Eq. (9) at $t = 0$.

Case 1. One-solitoff structure

If we take $g = \mathrm{e}^{4y-t} + 1$, from Eq. (8) we have

$$w = -\frac{4\mathrm{e}^{x+4y-t}}{(\mathrm{e}^{x} + \mathrm{e}^{4y-t} + 1)^2}, \tag{9}$$

which is the one-solitoff structure for Eq. (1). Fig. 1 shows Eq. (9).

Case 2. Multi-solitoff structure

If one takes $g = \cosh(4y - t) + 1$, then one obtains the two-solitoff structure

$$w = -\frac{4\mathrm{e}^{x}\sinh(4y - t)}{(\mathrm{e}^{x} + \cosh(4y - t) + 1)^2}, \tag{10}$$

which is shown in Fig. 2. Eq. (10) is a new type of solitoff structure, which is not reported in the literature to our knowledge.

Case 3. Dromion structures

Now, we take $g = \mathrm{sn}(4y - t/m) + 2$, where $\mathrm{sn}(\xi|m)$ is the Jacobi elliptic function, m the moduli of the elliptic function, then from Eq. (8) we can obtain

$$w = -\frac{4\mathrm{e}^{x}\mathrm{cn}(4y - t|m)\mathrm{dn}(4y - t|m)}{[\mathrm{e}^{x} + \mathrm{sn}(4y - t|m) + 2]^2}, \tag{11}$$

which is the oscillating dromion structure of Eq. (1). And its typical spatial structure is depicted in Fig. 3. As $m \to 1$, it follows from Eq. (11) that

$$w = -\frac{4\mathrm{e}^{x}\mathrm{sech}^2(4y - t|m)}{[\mathrm{e}^{x} + \tanh(4y - t|m) + 2]^2}, \tag{12}$$

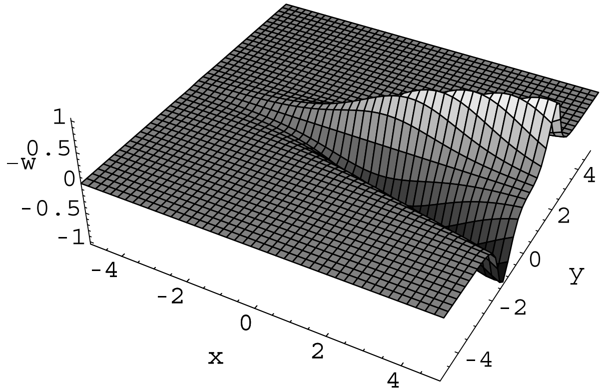

Fig. 2. The structure graph of Eq. (10) at $t = 0$.

which is the one-dromion structure of Eq. (1). Figure 4 illustrates Eq. (12). Eqs. (11) and (12) are new types of dromion structures, which are not reported previously.

The dromion and solitoff structures are interesting localized coherent ones (Boiti *et al.*, 1988; Fokas and Santini, 1989; Hietarinta and Hirota, 1990; Gilson, 1992; Chow, 1996) in higher-dimensional nonlinear physical models. In this paper, we have obtained some new types of dromion and solitoff structures, which are quite different from the basic dromion and solitoff structures (Boiti *et al.*, 1988; Chow, 1996; Fokas and Santini, 1989; Gilson, 1992; Hietarinta and Hirota, 1990), for the BS equation (1). It is worthy of mentioning that all these new solutions can

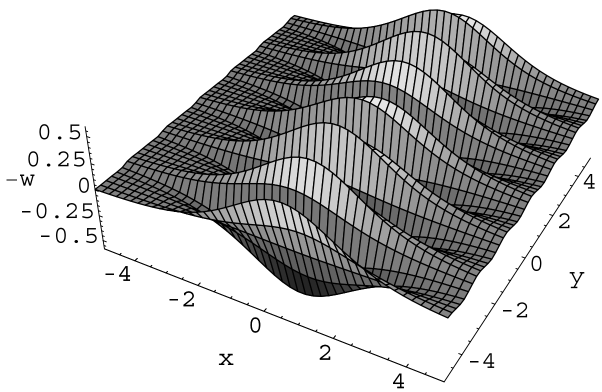

Fig. 3. The structure graph of Eq. (11) at $t = 0$.

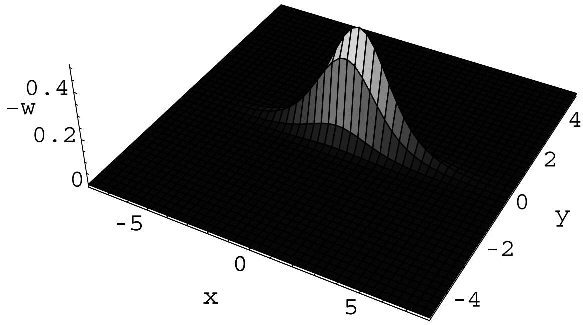

Fig. 4. The structure graph of Eq. (12) at $t = 0$.

propagate stably. Whether Eq. (1) possesses other localized coherent structures is worth studying further.

REFERENCES

Bogoyavlenskii, O. I. (1990). *Mathematics of the USSR-Izvestiya* **34**, 245.
Boiti, M., Leon, P. J., Martina, L., and Pempinelli, F. (1988). *Physics Letters A* **132**, 432.
Chow, K. W. (1996). *Journal of Physical Society of Japan* **65**, 1971.
Fokas, A. and Santini, P. M. (1989). *Physical Review Letters* **63**, 1329.
Gilson, C. R. (1992). *Physics Letters A* **161**, 423.
Hietarinta, J. and Hirota, R. (1990). *Physics Letters A* **145**, 237.
Schiff, J. (1992). *Painlevé Transcendents, Their Asymptotics and Physical Applications*, Plenum, New York.
Toda, K., Ju, Y. S., and Fukuyama, T. (1999). *Reports on Mathematical Physics* **44**, 247.